Revision Exercise for Home Test 2

Question 1. Show by definition that

(a) $\lim_{x\to 3} \dfrac{x^3-9}{2x^2-9} = 2$,

(b) $\lim_{x\to 1-} \dfrac{x}{1-x} = \infty$.

Question 2. Show that the function $f(x) = 1/x$ is uniformly continuous on $[1,\infty)$, but it is not uniformly continuous on $(0,\infty)$.

Question 3. Let $f : [0,\pi/2] \to \mathbb{R}$ be defined by $f(x) = \sup\{x^2, \cos x\}$. Show that f has an absolute minimum. Moreover, show that if f attains its minimum at x_0, then x_0 is a solution to the equation $\cos x = x^2$.

Question 4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous periodic function with period $p > 0$. i.e., $f(x+p) = f(x)$ for all $x \in \mathbb{R}$. Show that

(a) f has an absolute maximum.

(b) f is uniformly continuous on $\mathbb{R}$.

Question 5. Let $f : \mathbb{R} \to \mathbb{R}$ be uniformly continuous on $\mathbb{R}$ with $f(0) = 0$. Prove that there exists some $C > 0$ such that

$$|f(x)| \le 1 + C|x|, \quad \forall x \in \mathbb{R}.$$

(Hint: You may apply the **Well-Ordering Property of** $\mathbb{N}$.)